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Indian Statistical Institute
Bangalore Centre
B.Math (Hons.) Second Year 2009-2010
Second Semester
Mid-Semester Examination
Statistics II

22.02.10 S. Bagchi Time :- 3 hours

Answer as much as you can. The maximum you can score is 60
The notation used have their usual meaning unless stated otherwise.
State clearly the results that you assume.

1. (a) Define a sufficient statistics for a parameter. State factorization theorem.

   (b) Suppose $X_1, X_2, \cdots X_n$ is a random sample from a $N(\mu, \sigma^2)$ population. Show that $T(X) = (T_1 = \bar{X}, T_2 = \sum_{i=1}^{n} X_i^2)$ is jointly sufficient for (μ, σ^2).

   (c) When is a statistic called minimal sufficient ?

   Suppose $f(\underset{\sim}{x}, \theta)$ is the pdf of a sample $\underset{\sim}{X} = (X_1, X_2, \cdots X_n)$. Suppose there exists a function $T(\underset{\sim}{X})$ such that, for every two sample points $\underset{\sim}{x}$ and $\underset{\sim}{y}$, the ratio $f(\underset{\sim}{x}, \theta)/f(\underset{\sim}{x}, \theta)$ is independent of θ if and only if $T(\underset{\sim}{x}) = T(\underset{\sim}{x})$. Then show that $T(\underset{\sim}{X})$ is a minimal sufficient statistic for θ.

   Is $T(X)$ of question (b) minimal sufficient for (μ, σ^2) ? Give reason.

   (d) Define (a) ancillary statistic and (b) a complete statistic.

   (e) Suppose $X_1, X_2, \cdots X_n$ is a random sample from a uniform $(\theta, \theta+1)$ population. Let $T_1(\underset{\sim}{X}) = X_{(1)}$ and $T_2(\underset{\sim}{X}) = X_{(n)}$. Find the joint density of $T = (T_1, T_2)$. Find an ancillary statistic which is a function of T. What can you say about the completeness of T ?

   [(3+3)+ 4 + (3+5+3) + (2+2) + (4+3+2) = 34]

2. (a) Suppose $X_1, X_2, \cdots X_n$ is a random sample from one of the following populations. In each case find the maximum likelihood estimate (M.L.E.) of θ.

   (a) Poisson (θ), Uniform $(0, \theta)$.

   (b) If $\hat{\theta}$ is an M.L.E. of θ, then show that $h(\hat{\theta})$ is also an M.L.E. of $h(\theta)$ for any function h.

   [(3+3)+ 5 = 11]

3. (a) Show that $W(X)$ is the minimum variance unbiased estimator (M.V.U.E.) of its expected value if and only if $W(X)$ is uncorrelated with every unbiased estimators of 0.

   (b) Show that an M.V.U.E. is unique.

   (c) Suppose $T(X)$ is an complete sufficient statistic for a parameter θ and $S = \phi(T)$ has finite variance. Show that S is the unique M.V.U.E. of its expected value.

   (d) Suppose $X_1, X_2, \cdots X_n$ is a random sample from the Binomial $(1,\theta)$ population. Show that $T(\underset{\sim}{X}) = \sum_{i=1}^{n} X_i$ is complete sufficient for θ. Find an M.V.U.E for θ.

   [6 + 4 + 8 + (7+2) = 27]